Trig Functions

Warmup

1) $1 - \dfrac{\sin^2 x}{1 + \cos x} =$ $\quad\quad$ $\cos x$

2) $1 - \dfrac{\cos^2 x}{1 + \sin x} =$ $\quad\quad$ $\sin x$

3) $\dfrac{\sin x}{\cos x + 1} + \dfrac{\cos x - 1}{\sin x} =$ $\quad\quad$ 0

Law of Sines

7.1 - Law of Sines

Sine Ratios:

Acute triangle

Obtuse triangle
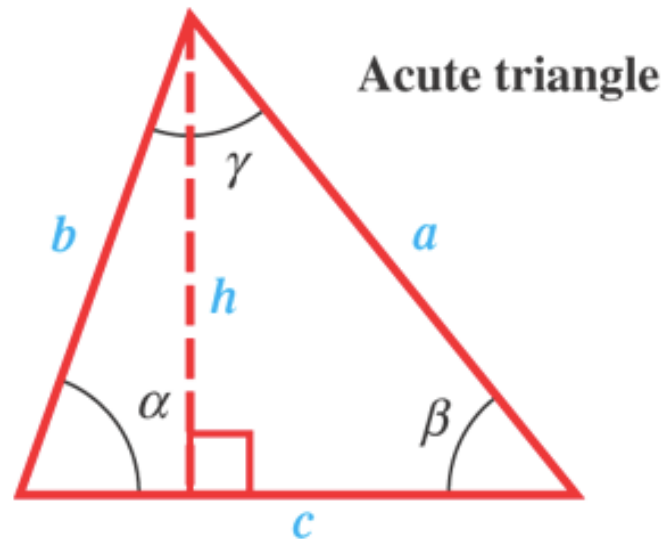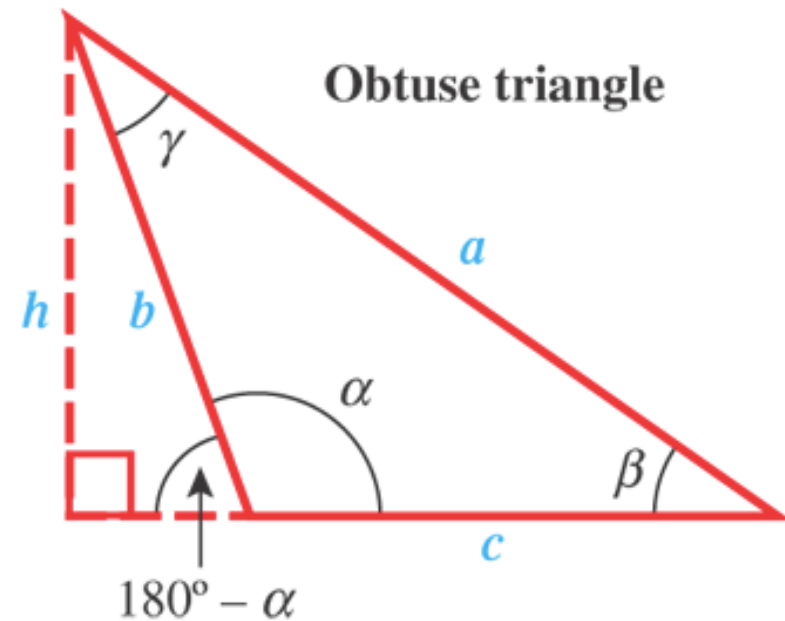
$$\sin \alpha = \frac{h}{b} \qquad \sin \beta = \frac{h}{a}$$

$$\sin(180° - \alpha) = \sin \alpha$$

$$\sin \alpha = \frac{h}{b} \qquad \sin \beta = \frac{h}{a}$$

SAME!!!

7.1 - Law of Sines
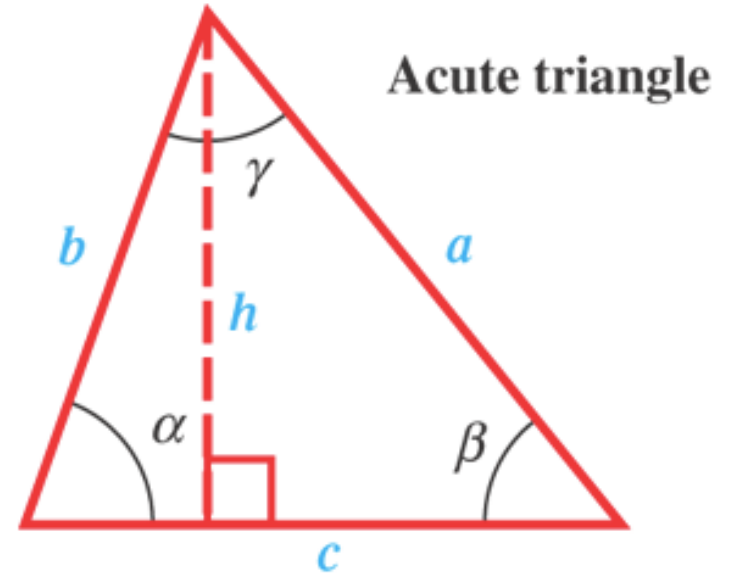
$$\sin \alpha = \frac{h}{b} \qquad \sin \beta = \frac{h}{a}$$

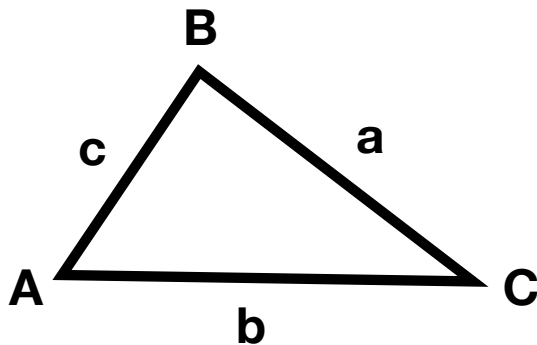$$h = b \sin \alpha \qquad h = a \sin \beta$$

$$b \sin \alpha = a \sin \beta$$

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b}$$

Acute triangle

$$\boxed{\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}}$$

Law of Sines

7.1 - Law of Sines

B

c

a

A

b

C

$$\frac{\sin \mathbf{A}}{\mathbf{a}} = \frac{\sin \mathbf{B}}{\mathbf{b}} = \frac{\sin \mathbf{C}}{\mathbf{c}}$$

Practice

1) A = 40˚, B = 30˚, b = 10 (AAS)

$C = 110°$

$a \approx 12.86$

$c \approx 18.79$

2) A = 32˚, b = 11, a = 17 (SSA)

$B \approx 20°$

$C \approx 128°$

$c \approx 25.28$

3) A = 64˚, a = 16, b = 17 (SSA)

$B \approx 72.74°$ $B \approx 107.26°$

$C \approx 43.26°$ $C \approx 8.74°$

$c \approx 12.20$ $c \approx 2.70$

Law of Cosines

7.2 Law of Cosines

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Find x

$$\cos A = \frac{x}{c}$$

$$c \cdot \cos A = x$$
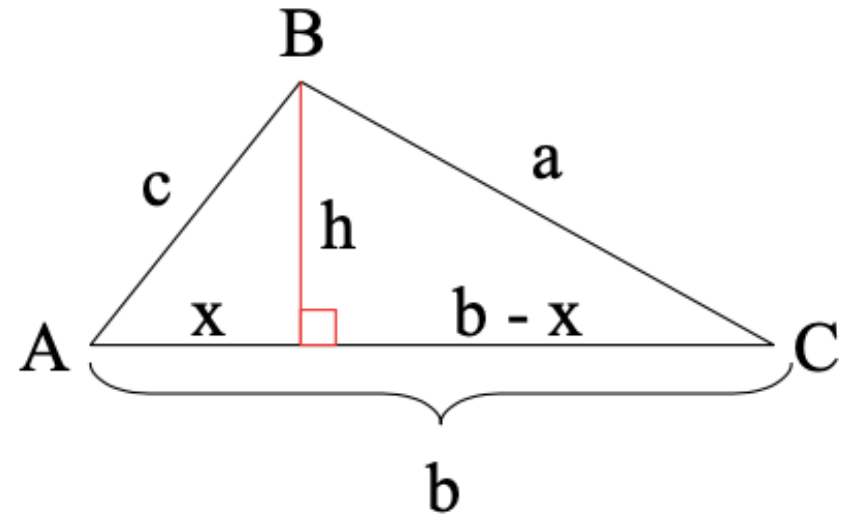
Pythagorean Theorem

$$a^2 = (b-x)^2 + h^2$$

$$a^2 = b^2 - 2bx + \boxed{x^2 + h^2}$$

$$a^2 = b^2 - 2b\boxed{x} + c^2$$

$$a^2 = b^2 - 2bc \cdot \cos A + c^2$$

$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cdot \cos A \\ b^2 &= a^2 + c^2 - 2ac \cdot \cos B \\ c^2 &= a^2 + b^2 - 2ab \cdot \cos C \end{aligned}}$$
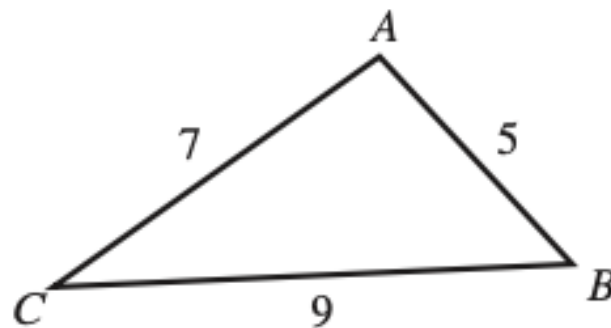
Law of Cosines

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$ **Law of Cosines**

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

Practice - Find the angle A

$A \approx 95.7°$

Area of a Triangle
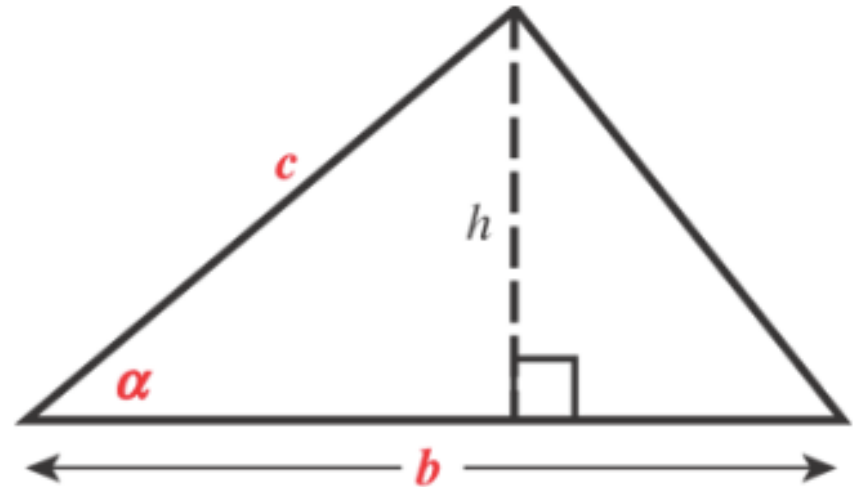
7.3 - Area of a Triangle

SAS Case

$$\sin \alpha = \frac{h}{c}$$

$$h = c \sin \alpha$$

$$A = \frac{1}{2}bh$$

$$\boxed{\mathbf{A} = \frac{1}{2}\mathbf{bc} \sin \alpha}$$

7.3 - Area of a Triangle

$$A = \frac{1}{2}bc \sin \alpha$$

Find the area of the triangle $a = 7.0$ feet, $b = 9.3$ feet, and $\gamma = 86°$.

$$A \approx 32.47 \ ft^2$$

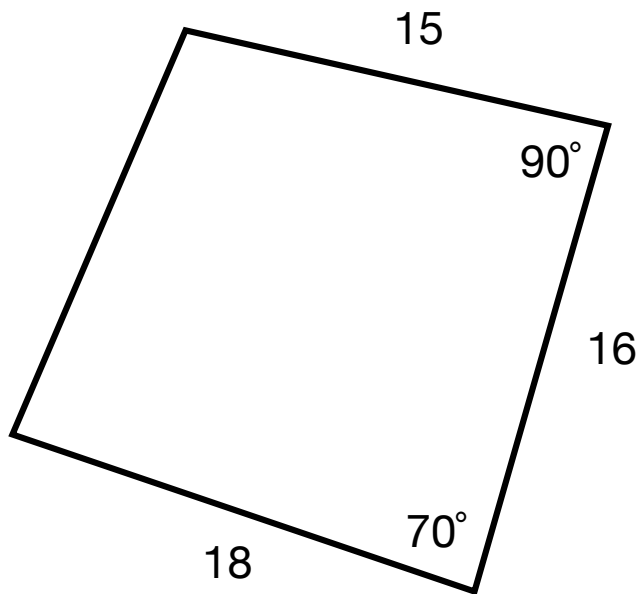Practice: Find the area of the triangle $a = 3.2$ meters, $c = 5.1$ meters, and $\beta = 49°$.

$$A \approx 6.2 \ m^2$$

7.3 - Area of a Triangle

$$A = \frac{1}{2}bc \sin \alpha$$

How do you find the area?

15

90°

16

70°

18

area ≈ 120 + 89.14 = 209.14
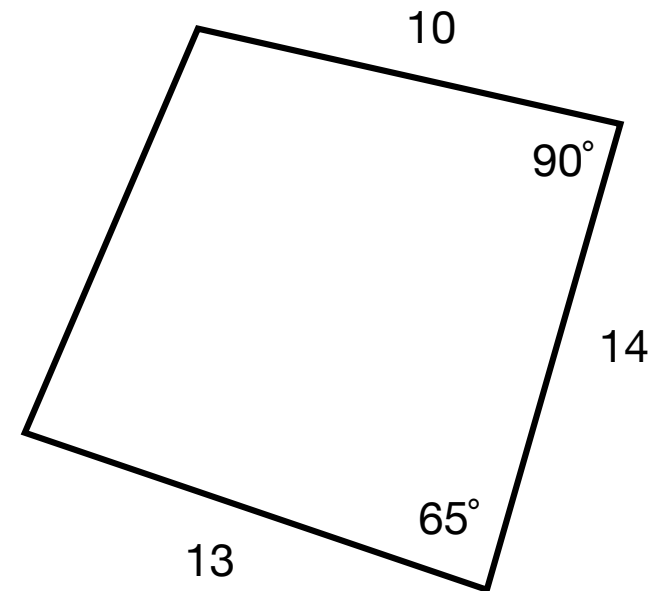
7.3 - Area of a Triangle

Practice - Find the area

$$A = \frac{1}{2}\mathbf{bc}\sin\alpha$$

1.

12

115°

90°

15

16

area ≈ 96 + 132.3 = 228.3

2.

10

90°

14

65°

13
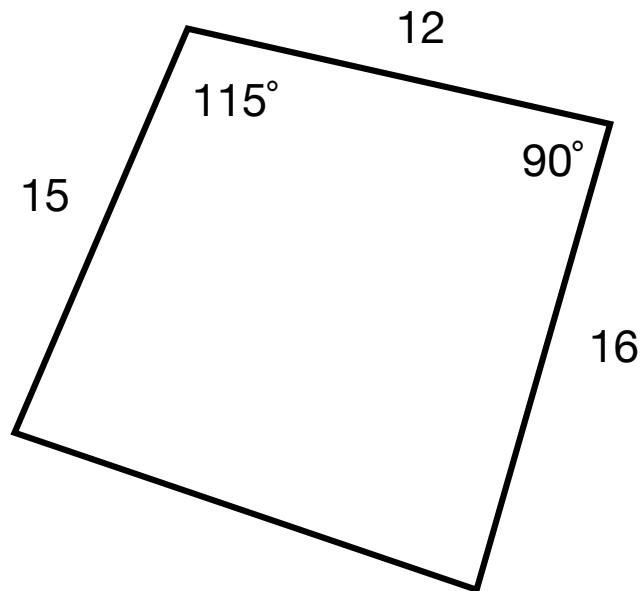
area ≈ 70 + 55 = 125

Heron's Formula
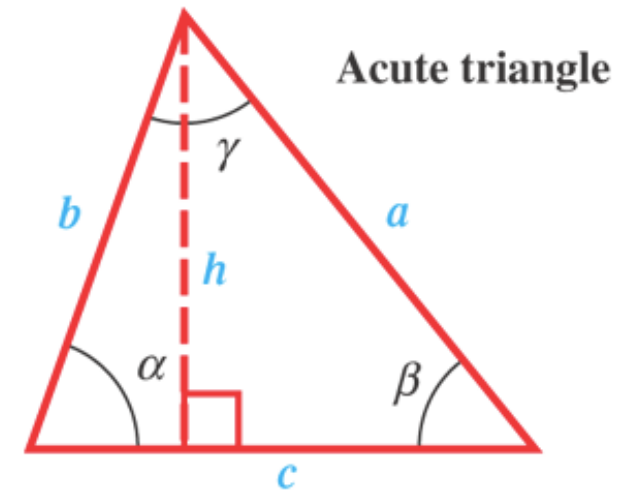
7.3 - Area of a Triangle

SSS Case

Over 2,000 years ago a mathematician (Heron) from Alexandria, Egypt, derived the following formula.

Acute triangle

Semiperimeter s

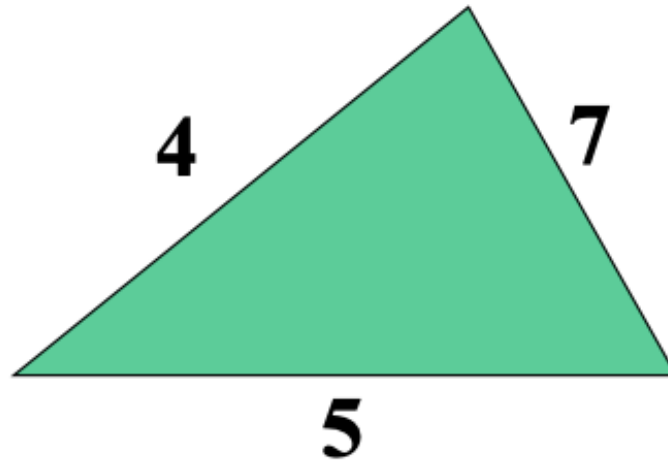$$s = \frac{1}{2}(a + b + c)$$

$$\boxed{A = \sqrt{s(s-a)(s-b)(s-c)}}$$

7.3 - Area of a Triangle

$$s = \frac{1}{2}(a + b + c) \qquad A = \sqrt{s(s - a)(s - b)(s - c)}$$

Find the area.

$$A = 4\sqrt{6} \approx 9.8 \; sq \; units$$

7.3 - Area of a Triangle

$$s = \frac{1}{2}(a + b + c)$$

$$\mathbf{A} = \sqrt{\mathbf{s(s-a)(s-b)(s-c)}}$$

Practice

1. Find the area of the triangle $a = 5$, $b = 6$, and $c = 9$.

$$A = 10\sqrt{2} \approx 14 \; sq \; units$$

2. Find the area of the triangle $a = 3$, $b = 5$, and $c = 6$.

$$A = 2\sqrt{14} \approx 7.5 \; sq \; units$$

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